Nigerian Stock Index: A Search for Optimal GARCH Model using High Frequency Data

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This paper attempts to fit the best Generalized Autoregressive Conditional Heteroscedastic (GARCH) model for All Share Index (ASI) of Nigerian Stock Exchange (NSE) returns. A search is made on various GARCH variants specified on the assumptions of stationarity and asymmetry. Fractionally integrated types are also considered to capture the possibility of return series having property of long range dependency. The parameter estimations are carried out on the assumptions of normality and non-normality of GARCH innovations, with models and forecasts evaluated using information criteria and loss functions respectively. Under normality assumption, Hyperbolic GARCH (HYGARCH(1,d,1)) model is selected and Integrated GARCH (IGARCH(1,1)) and Fractionally Integrated Exponential GARCH (FIEGARCH(1,d,1)) models selected under the Student t and Generalized Error Distributions. Of these three models, HYGARCH(1,d,1) is the overall best model.

Key Words: All Share Index, Daily stock prices, GARCH, Nigerian Stock Exchange

JEL Classification: C22, G14, G15

1.0 Introduction

The Nigerian Stock Exchange (NSE) came into existence in the Nigerian capital market in 1960. It was formerly known as the Lagos Stock Exchange (LSE). Under the platform of NSE, local and foreign stocks are traded and all the stocks are used to compute the All Share Index (ASI). In May, 2001, the ASI crossed the 10,000 point mark and stood at 10,153.8 at the end of the month. The NSE houses a large chunk of the nation's wealth and has continued to be the major discourse of various studies since the advent of the global crisis. Therefore, there is need to study the pattern or stochastic process underlying these stocks. Capital Asset Pricing Model (CAPM) was initially applied and the model relates the expected price of an asset to its risk measured by the variance of the asset's historical rate of return relative to its asset class (Sharpe, 1964). In 1982, Engle R.F. proposed an Autoregressive

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Conditionally Heteroscedastic (ARCH) model to capture the volatility in asset returns and through this model, variants of generalized forms of the model have been proposed and applied in the area of financial time series.

The recent global financial crisis has gingered researchers towards studying the economy particularly in the developing world. In the developed world, this has been considered over the years except for recent applications of high frequency data. Stocks, for example are investigated up to daily frequency using the business days only. In Nigeria, researches are few on stocks, and the few ones have applied monthly frequency data. Among these is the work of Salisu (2012) which considered monthly ASI series from 1985:01 to 2010:12 for some symmetric and asymmetric Generalized Autoregressive Conditionally Heteroscedastic (GARCH) variants and obtained Exponential GARCH (EGARCH) as the optimal model for the series. The estimation was also performed based on the assumption of normal distribution of the innovations for GARCH model. The work applied only the stationary symmetric and asymmetric variants of GARCH models, whereas there are other variants that consider fractional integration and nonstationarity of the conditional volatility specifications.

This paper therefore considers estimating daily ASI using the stationary, nonstationary and fractionally integrated GARCH variants for both symmetric and asymmetric forms. The rest of the paper is structured as follows: Section 2 reviews the variants of GARCH models applied in this paper. Section 3 presents the data and empirical results. Section 4 gives the concluding remarks.

2.0 Variants of the GARCH model

Since the seminar article of Engle, R.F. in 1982, different GARCH specifications have been proposed in the literature. The bases of proposition of the models are the ARCH (p) of Engle (1982) and GARCH(p,q) of Bollerslev (1986). These variants are classified into stationary symmetric (ARCH and GARCH); nonstationary symmetric Integrated GARCH (IGARCH); stationary asymmetric (EGARCH, Glosten Jaganathan and Runkle (GJR) and Asymmetric Power ARCH (APARCH)) and Fractionally Integrated (FI) specifications (FIGARCH, FIEGARCH, FIAPARCH and Hyperbolic

GARCH (HYGARCH)). There are other specifications in the literature but these ones have gained popularity in terms of their applications.

Initially, the asset prices are transformed into log return series, r_t given by

$$r_t = \log X_t - \log X_{t-1} = \log\left(\frac{X_t}{X_{t-1}}\right) = \log\left(1 + \frac{X_t - X_{t-1}}{X_{t-1}}\right) \quad (1)$$

where X_t is ASI for day t. Then, Autoregressive (AR) model

$$r_t = c_0 + \sum_{i=1}^{m} c_i r_{t-i} + \varepsilon_t \quad (2)$$

is estimated for the return series. The parameters c_0 and c_i are the constant and AR parameters to be estimated and m is the order of autoregression. The best order of the model is chosen based on Information Criteria. The residuals ε_t in (2) are described as $\varepsilon_t = \sigma_t z_t$ where σ_t follows the conditional volatility models described henceforth and the GARCH innovations $z_t = \varepsilon_t / \sigma_t$ follows normal distribution, Student t distribution or Generalized Error Distribution (GED). [1]

Engle (1982) therefore proposed modelling the variance (σ_t^2) of residuals ε_t with ARCH(q) model

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 = \omega + \alpha(L)\varepsilon_t^2 \quad (3)$$

where the parameters $\omega > 0$, $\alpha_i \geq 0$ for $i = 1, \ldots, p$. The polynomial $\alpha(L) = \alpha L = \sum_{i=1}^{p} \alpha_i L^i$ of order p is defined for the ARCH parameter. Bollerslev (1986) generalized Engle's model by including lags of unconditional variance in the model as given as,

[1] Details of these distributions are discussed in the latter part of this paper

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i \sigma_{t-j}^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \qquad (4)$$

where $\beta_i \geq 0$ for $j = 1,...,q$, $\beta(L) = \beta L = \sum_{j=1}^{q} \beta_j L^j$ is the polynomial of order q defined for the GARCH parameter and other parameters remain as defined above. Equation (4) can be represented as Autoregressive Moving Average (ARMA(p,q)) process defined as,

$$\varepsilon_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \left\{ \left(\sum_{j=1}^{q} \beta_i \sigma_{t-j}^2 \right) + V_t - \sum_{j=1}^{q} \beta_j V_{t-j} \right\} \qquad (5)$$

In compact form, (5) is re-expressed as,

$$\Phi(L)(1-L)\varepsilon_t^2 = \omega + (1-\alpha(L))V_t \qquad (6)$$

or,

$$\sigma_t^2 = \omega + (1 - \beta(L) - \Phi(L)(1-L))\varepsilon_t^2 + \beta(L)\sigma_t^2 \qquad (7)$$

where $\Phi(L) = 1 - \sum_{i=1}^{p} \phi_i L^i$ and L is the backward shift operator. From (4), there is a second order stationarity if the roots of $\alpha(L) + \beta(L) = 1$ lie outside the unit circle. Since the estimate of $\alpha(L) + \beta(L)$ is always very close to unity, this motivated the development of IGARCH(p,q) model of Engle and Bollerslev (1986) which is

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \qquad (8)$$

The first asymmetric model proposed in Nelson (1991) is EGARCH(p,q) which has the form

$$\log\sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right)\right| + \sum_{j=1}^{q}\beta_j\log\sigma_{t-j}^2 + \sum_{k=1}^{r}\gamma_k\left(\frac{\varepsilon_{t-k}}{\sigma_{t-k}}\right) \quad (9)$$

with the parameters as defined in the GARCH model in (4) except for $\gamma_k \neq 0$, which allows for the asymmetric effect. The component $E\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right)$ in the model is given as $E\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| \approx \sqrt{\frac{2}{\pi}}$ under normally distributed innovations; $E\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| \approx \Gamma\left(\frac{v+1}{2}\right)^2 \frac{\sqrt{v-2}}{\sqrt{\pi}(v-1)\Gamma(v/2)}$ for the Student *t* distribution and $E\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| \approx \lambda 2^{v-1}\frac{\Gamma(2v^{-1})}{\Gamma(v^{-1})}$ for the Generalized Error Distribution (GED). Ding, *et al.* (1993) introduced the APARCH (*p*, *q*) model,

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p}\alpha_i\left(\left|\varepsilon_{t-i}\right| - \gamma_i\varepsilon_{t-i}\right)^\delta + \sum_{j=1}^{q}\beta_j\log\sigma_{t-j}^\delta \quad (10)$$

where the asymmetric parameter $-1 < \gamma_i < 1 (i = 1,..., p)$, δ is the non-negative Box-Cox power transformation of the conditional standard deviation process and asymmetric absolute innovations. This power parameter is estimated along with other parameters in the model. Glosten *et al.* (1993) also proposed the GJR (*p*, *q*) model,

$$\sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i\varepsilon_{t-i}^2 + \sum_{i=1}^{p}\left[\gamma_i d\left(\varepsilon_{t-i} < 0\right)\varepsilon_{t-i}^2\right] + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2 \quad (11)$$

where $\gamma_i (i = 1,..., p)$ are the asymmetric parameter and $d(.)$ is the indicator function defined such that $d(\varepsilon_{t-i} < 0) = 1$ if $\varepsilon_{t-i} < 0$ and $d(\varepsilon_{t-i} > 0) = 0$ if $\varepsilon_{t-i} > 0$.

2.1 The variants of Fractionally Integrated GARCH models

Initially, the Fractionally Integrated GARCH (FIGARCH) model was proposed and variants of these models, which include both symmetric and asymmetric types, have emerged. The FIGARCH (p,d,q) model was motivated by the works of Granger (1980) and Granger and Joyeux (1980) who both proposed a time series model,

$$\Phi(L)(1-L)^d X_t = \Theta(L)\varepsilon_t \tag{12}$$

where $\Phi(L)$ and $\Theta(L)$ are polynomials of degrees p and q defined as $\Phi(L) = 1 - \sum_{i=1}^{p} \phi_i L^i$, $\Theta(L) = 1 + \sum_{j=1}^{q} \theta_j L^j$ and ε_t has a zero mean and constant variance and as well serially uncorrelated. The parameter d is set such that $|d| < 0.5$, hence X_t is covariance stationary and the process in (12) is termed the Autoregressive Fractionally Integrated Moving Average (ARFIMA) process of order (p,d,q). The fractional differencing operation involved in ARFIMA(p,d,q)process led to the introduction of many fractional integrated time series models which include GARCH models. The fractional differencing operator $(1-L)^d$ is expressed in terms of hypergeometric function as

$$(1-L)^d = F(-d,1;1;L) = \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)} L^j = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j , \tag{13}$$

where $\Gamma(.)$ is the gamma function and the hypergeometric function,

$$F(a,b;c;z) = \sum_{j=0}^{\infty} \frac{(a)_j (b)_j}{(c)_j} \frac{z^j}{j!} \quad \text{with} \quad (b)_j = \Pi_{i=0}^{j-1}(b+1) \quad \text{and} \quad (b)_0 = 1,$$

simplified as, $F(a,b;c;z) = 1 + \frac{a \cdot b}{1 \cdot c} z + \frac{a(a+1) \cdot b(b+1)}{1 \cdot 2 \cdot c(c+1)} z^2 + \ldots$. The inverse of the expansion of the fractional differencing operator in (13) is,

$$\left(1-L\right)^{-d}=\sum_{j=0}^{\infty}\frac{\Gamma\left(j+d\right)}{\Gamma\left(j+1\right)\Gamma\left(d\right)}L^{j} \qquad (14)$$

Using the backshift operator in (14) on the GARCH model in (4), this results in the FIGARCH(p,d,q) model of Baillie *et al*. (1996) which is specified as,

$$\sigma_t^2=\omega+\left(1-\beta\left(L\right)-\Phi\left(L\right)\left(1-L\right)^{d}\right)\varepsilon_t^2+\beta\left(L\right)\sigma_t^2 \qquad (15)$$

and this is strictly stationary and ergodic for $0\le d\le 1$ and covariance stationary for $\left|d\right|<0.5$. For $d=1$, the model collapses to IGARCH$\left(p,q\right)$ and this as well collapses to GARCH$\left(p,q\right)$ model when $d=0$. The expansion of (13) in (15) leads to,

$$\sigma_t^2=\omega+\left(1-\beta\left(L\right)-\Phi\left(L\right)\sum_{j=0}^{\infty}\frac{\Gamma\left(j-d\right)}{\Gamma\left(j+1\right)\Gamma\left(-d\right)}L^{j}\right)\varepsilon_t^2+\beta\left(L\right)\sigma_t^2 \qquad (16)$$

The FIGARCH $\left(p,d,q\right)$ model explained earlier the persistence, fat-tailed and volatility clustering in the series. However, there are limitations in its structure. Its variance structure depends only on the sign of innovations ε_t which is contrary to the empirical behaviour of stock market prices which allows for the leverage effect. Changes in stock returns give negative correlation with changes in volatility, that is, volatility rises in response to negative innovations (bad news) and to fall in response to positive innovations (good news) (Black, 1976). This is the leverage effect. The FIGARCH $\left(p,d,q\right)$ is said to be symmetric since only the magnitude of ε_t determines the variance σ_t^2. The asymmetric representations of the models allows for both positivity (good news) and negativity (bad news) of the innovations to determine the variance just as in the previously discussed EGARCH, GJR and APARCH models.

Bollerslev and Mikkelsen (1996) therefore applied EGARCH model in a fractionally integrated specification and proposed the first variant of FIGARCH model defined as the Fractional Integrated Exponential GARCH (FIEGARCH(*p*,*d*,*q*)) model given as,

$$\log\sigma_t^2=\omega+\Phi(L)^{-1}(1-L)^d(1+\alpha(L))\left[\gamma_1\frac{\varepsilon_{t-1}}{\sigma_{t-1}}+\gamma_2\left(\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}-E\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right)\right|\right)\right]+\beta(L)\log\sigma_t^2$$

$$=\omega+\Phi(L)^{-1}(1+\alpha(L))\left[1+\sum_{j=1}^{\infty}\frac{\Gamma(j+d)}{\Gamma(j+1)\Gamma(d)}L^j\right]\left[\gamma_1\frac{\varepsilon_{t-1}}{\sigma_{t-1}}+\gamma_2\left(\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}-E\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right)\right|\right)\right]+\beta(L)\log\sigma_t^2 \quad (17)$$

Tse (1998) developed the Fractionally Integrated APARCH (FIAPARCH(p,d,q)) model as a follow up research work on Ding *et al.* (1993) by introducing fractional integration. The model is given as,

$$\sigma_t^\delta=\omega+\left(1-(1-\beta(L))^{-1}\Phi(L)^{-1}(1-L)^d\right)\left(|\varepsilon_t|-\gamma\varepsilon_t\right)^\delta+\beta(L)\sigma_t^\delta$$

$$=\omega+\left(1-(1-\beta(L))^{-1}\Phi(L)^{-1}\left(1+\sum_{j=1}^{\infty}\frac{\Gamma(j+d)}{\Gamma(j+1)\Gamma(d)}L^j\right)\right)\left(|\varepsilon_t|-\gamma\varepsilon_t\right)^\delta+\beta(L)\sigma_t^\delta \quad (18)$$

where $\beta(L)$ and $\Phi(L)$ are as defined in earlier part of this work. The d is the fractional difference parameter, $\delta\geq 0$ and assumes values 1 or 2. The asymmetric parameter is set as $-1<\gamma<1$ where at $\gamma>0$, negative shocks give rise to higher volatility than positive shocks. At $\gamma<0$, positive shocks give rise to higher volatility than negative shocks (Tse, 1998).

Davidson (2004) proposed the Hyperbolic GARCH (HYGARCH(p,d,q)) given as

$$\sigma_t^2=\omega+\left(1-\beta(L)-\Phi(L)\left(1+a\left((1-L)^d-1\right)\right)\right)\varepsilon_t^2+\beta(L)\sigma_t^2$$

$$=\omega+\left(1-\beta(L)-\Phi(L)\left(1+a\left(\sum_{j=1}^{\infty}\frac{\Gamma(j+d)}{\Gamma(j+1)\Gamma(d)}L^j-1\right)\right)\right)\varepsilon_t^2+\beta(L)\sigma_t^2 \quad (19)$$

where a is the weight factor and at $a=1$, the HYGARCH model nests the FIGARCH model. The weight factor is taken as $\log a$ in some other model specifications.

2.2 Distributional forms and Estimation of GARCH models

Estimation of GARCH models is based on the assumption of normality, Students t and Generalized Error Distributions (GED) [2] for the innovations series ε_t. The log-likelihood from the normal distribution is

$$l_t = -\frac{1}{2}\left[N\log(2\pi) + \sum_{t=1}^{N}\frac{\varepsilon_t^2}{\sigma_t^2} + \sum_{t=1}^{N}\log\sigma_t^2 \right] \qquad (20)$$

and with $\varepsilon_t = \sigma_t z_t$ where $z_t = \varepsilon_t / \sigma_t$ is the GARCH time series innovations and N is the sample size of the time series. For the Student *t*-distribution, we have

$$l_t = -\frac{1}{2}\left\{ N\log\left(\frac{\pi(v-2)\Gamma(v/2)^2}{\Gamma((v+1)/2)^2} \right) + \sum_{t=1}^{N}\log\sigma_t^2 + (v+1)\sum_{t=1}^{N}\log\left[1 + \frac{\varepsilon_t^2}{\sigma_t^2(v-1)} \right] \right\} \qquad (21)$$

where v is the degrees of freedom to be estimated and $\Gamma(.)$ is the gamma function. For GED, it is

$$l_t = -\frac{1}{2}\left\{ N\log\left(\frac{\Gamma(v^{-1})^3}{\Gamma(3v^{-1})(v/2)^2} \right) + \sum_{t=1}^{N}\log\sigma_t^2 + (v+1)\sum_{t=1}^{N}\left(\frac{\Gamma(3v^{-1})\varepsilon_t^2}{\sigma_t^2\Gamma(v^{-1})} \right)^{v/2} \right\} \qquad (22)$$

where v is the tail thickness parameter. These log-likelihood functions in (20), (21) and (22) are simplified using MaxSA algorithm of Goffe *et al.* (1994) implemented in GARCH program of Laurent (2007) and Laurent and Peters (2006).

2.3 Evaluation of GARCH models

The GARCH variants will be evaluated by Akaike's Information Criterion (AIC) (Akaike, 1974) and Schwarz Bayesian Information Criterion (SBIC) (Schwarz, 1978), even though the statistical properties of the criteria in the GARCH context are yet to be known. The two criteria are given as,

$$AIC = -2N^{-1}l_t(\Theta) + 2N^{-1}k \qquad (23)$$

[2] Extensive literature review on GARCH probability distributional assumptions, see Bollerslev (1987) for Student *t* distribution and Nelson (1991) for GED

and

$$SBIC = -2N^{-1}l_t\left(\Theta\right) + 2kN^{-1}\ln\left(N\right) \qquad (24)$$

where $l_t\left(\Theta\right)$ is the maximum likelihood function conditioned on the parameter set Θ.

2.4 Evaluation of Forecasts

It is not an easy task to generate out-of-sample forecasts for GARCH models or even forecasts of nonlinear time series. In that case, only the in-sample-forecasts performances of the models will be examined. Comparing the forecast generated from models by the usual Mean Square Forecasts Error (MSFE) and Mean Absolute Forecast Error (MAFE) are common in the literatures. More recently, loss functions are used and these give equivalent results to the naive methods of evaluating forecasts. The Squared Error (SE) and Absolute Error (AE) loss functions have been used in Brooks and Persand (2003). The Heteroscedasticity-Adjusted Squared Error (HASE) and Heteroscedasticity-Adjusted Absolute Error (HAAE) loss functions were applied in Andersen *et al.* (1999). The Logarithmic Error (LE) loss function was used in Saez (1997) and the last one, the Gaussian Likelihood (GL) loss function was used in Bollerslev *et al.* (1994).

Denoting the forecasting variance over a τ–day period, the in-sample conditional forecasts variance is then given by $\hat{\sigma}_{t+1}^{2(\tau)}$ for a period of τ days, from $t+1$ to $t+\tau$ depending on the size of the time series. Since the actual variance for a period of τ business days from $t+1$ to $t+\tau$ is not observed, we therefore apply a proxy measurement for the actual volatility. The squared returns r_t^2 is used as proxy to measure daily volatility in the log returns these are generated for $i = t + 1 \ldots t + \tau$ days. This proxy measure is unbiased and is given by,

$$r_{t+i}^2 = \tilde{\sigma}_{t+1}^{2(\tau)} \qquad (25)$$

where r_t is the log-return series defined earlier in (1). The in-sample mean loss functions are then given as,

$$\overline{SE} = 1/\tau \sum_{i=1}^{\tau} \left(\hat{\sigma}_{t+1}^{2(\tau)} - r_{t+i}^{2} \right)^{2}; \tag{26}$$

$$\overline{AE} = 1/\tau \sum_{i=1}^{\tau} \left| \hat{\sigma}_{t+1}^{2(\tau)} - r_{t+i}^{2} \right|; \tag{27}$$

$$\overline{HASE} = 1/\tau \sum_{i=1}^{\tau} \left(1 - \left(r_{t+i}^{2} / \hat{\sigma}_{t+1}^{2(\tau)} \right) \right)^{2}; \tag{28}$$

$$\overline{HAAE} = 1/\tau \sum_{i=1}^{\tau} \left| 1 - \left(r_{t+i}^{2} / \hat{\sigma}_{t+1}^{2(\tau)} \right) \right|; \tag{29}$$

$$\overline{LE} = 1/\tau \sum_{i=1}^{\tau} \left[\log \left(r_{t+i}^{2} / \hat{\sigma}_{t+1}^{2(\tau)} \right) \right]^{2} \text{ and} \tag{30}$$

$$\overline{GL} = 1/\tau \sum_{i=1}^{\tau} \left[\log \left(\hat{\sigma}_{t+1}^{2(\tau)} \right) + \left(r_{t+i}^{2} / \hat{\sigma}_{t+1}^{2(\tau)} \right) \right] \tag{31}$$

The smaller the estimates of these loss functions in a particular GARCH model as compared with another model, the better the former forecasting model.

3.0 The Data, Empirical Analysis and Results

The data used in this paper are the daily All Share Index (ASI) of Nigerian Stock Exchange from January 2007 to December 2011 covering 1231 data points including business days and excluding public holidays. The time plot of the ASI is displayed in Figure 1.

We can see that ASI has undergone series of bull and bear phases as well as breaks observed in the time plot. The sampled data range (2007-2011) started with 33,163.94, peaked on 5 March 2008 at 66,371.20 and this crashed to 19,803.60 on 26 March 2009 and later recovered. Since then, ASI kept on galloping and the value as at 30 December 2011 was 20,730.63. It is obvious to see that Nigeria is yet to recover from the global financial crisis.

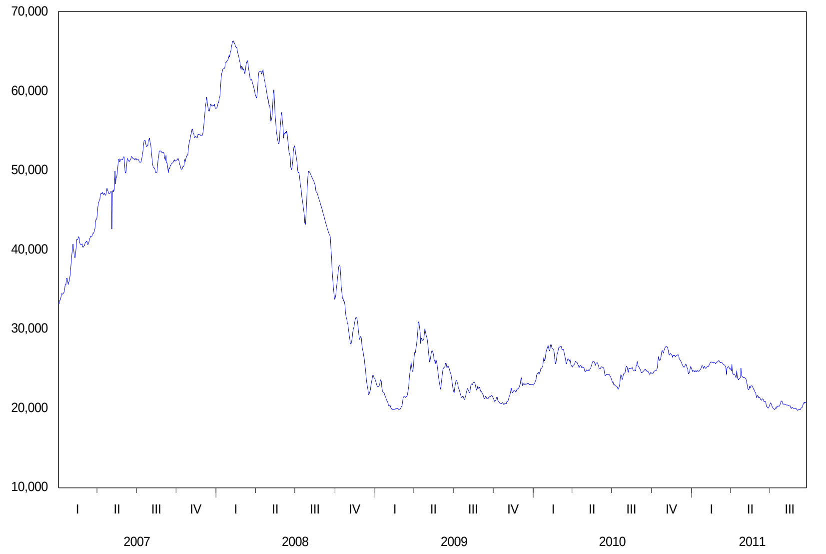

Figure 1: Time Plot of Daily Nigerian All Share Index (2007-2011)

Table 1: Descriptive Statistics of the Log-return, Absolute and Squared return series of ASI

	Log-return	**Absolute log-return**	**Squared log-return**
Mean	0.003724	-0.000166	0.000029
Median	0.002524	-0.000048	0.000006
Maximum	0.045563	0.045127	0.002076
Minimum	0.00000	-0.045563	0.0000000
Std. Dev.	0.00392	0.005405	0.000095
Skewness	3.03716	0.05951	16.16903
Kurtosis	24.2362	11.6205	336.4362
Jarque-Bera	25023.8***	3812.4***	5756231.0***

***significant at 1%

Further exploration into the data using the log-returns, absolute and squared log-returns of ASI gives the descriptive measures in Table 1 below. The absolute returns have been computed as $\left(|r_t|\right)$ and the squared returns as $\left(r_t^2\right)$ series.

Table 2: Best estimated GARCH variants for ASI Log-returns under the distributional assumptions

AR parameters	HYGARCH(1,d,1)-Normal	IGARCH(1,1)-*t*	FIEGARCH(1,d,1)-GED
c_0	-0.000196	-0.000299	-0.000555**
c_1	0.499931***	0.554708***	0.585501***
w	0.229039***	0.036945***	-114371.1***
α_1	0.790839***	0.446021***	0.088750***
β_1	0.575080***	0.553979***	0.030308***
γ_1			0.024213***
γ_2			0.572866***
d	-1.008912***		0.528539***
$\log a$	-5.783100***		
$d.f.$		3.437182***	1.079983***
Model Evaluation Criteria			
Log-lik.	4991.1	5103.06	5093.63
AIC	-8.091071	-8.276077	-8.254269
SBIC	-8.062	-8.255312	-8.216893
Skewness	-1.1794	-2.7506***	-2.1438***
Ex.Kurtosis	15.564	43.511***	32.841***
JB	12720	98737.0***	56309.0***
-ARCH(1) test	5.1596	0.45573	0.019115
ARCH(5) test	1.0996	0.15026	0.0.05518
ARCH(10) test	0.5968	0.08764	0.070444
Model Performance Using Statistical Loss Functions			
SE	9.59E-09	1.66E-08	5.61E-06
AE	3.11E-05	3.48E-05	9.61E-05
HASE	16.868392	37.483913	38.788271
HAAE	1.273677	1.187016	1.325937
LE	2.919682	2.9853	2.870448
GL	-3.475832	1.66E-08	5.61E-06

***significant at 1%, **significant at 5%, *significant at 10%.

The mean value computed in each return series is approximately zero, particularly in absolute and squared log-returns and this implies low volatility in the ASI. The return series are rightly skewed as indicated by positive estimates of skewness. Based on the estimates of skewness and kurtosis, it is clear that the returns series do not follow normal distribution.

In the search for the best GARCH variants for the ASI log return series, we first carried out the estimation of different GARCH variants considered in this paper on each of the distributions (Normal distribution, Student *t*-distribution and Generalized Error Distribution) with log-likelihood stated in (23), (24) and (25) respectively. That is the estimated models were the GARCH, EGARCH, APARCH, GJR, IGARCH, FIGARCH, FIEGARCH, FIAPARCH and HYGARCH. In the case of Normal distribution assumption, the best volatility model, based on minimum information criteria (AIC and SBIC) and loss functions is HYGARCH(1,d,1) model. For the Student *t*-distribution, the best model is IGARCH(1,1) while FIEGARCH(1,d,1) is the best model for Generalized Error Distribution. As summarized in Table 2, IGARCH(1,1) is the overall best model followed by FIEGARCH(1,d,1) model.

4.0 Concluding Remarks

In this paper, variants of GARCH models have been considered on Nigerian stocks using All Share Index (ASI) as proxy. The models were specified based on stationarity, fractional integration and nonstationarity for symmetric and asymmetric types. The three distributional assumptions of GARCH considered were Normal, Student *t* and Generalized Error Distributions and HYGARCH(1,*d*,1), IGARCH(1,1) and FIEGARCH(1,*d*,1) respectively emerged as the best models under each GARCH distribution. The selection of the best model for each distribution has been based on the minimum information criteria and loss functions. Of the three models selected for each of the GARCH distributions, HYGARCH(1,d,1) model is the overall best.

This work can be extended by (i) identifying the "bull" and "bear" stock market phases for the ASI and (ii) use apply the returns in each market phase on the variants of GARCH models considered in this work and compare the results with that obtained in this work.

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